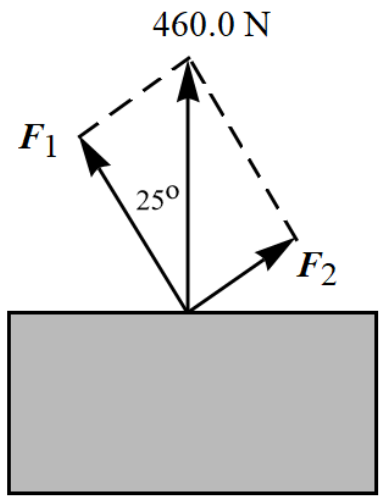
Problem 1.65

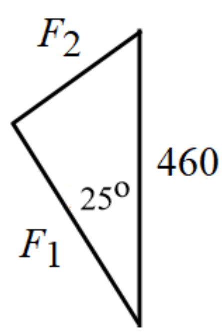
Two workers pull horizontally on a heavy box, but one pulls twice as hard as the other. The larger pull is directed at 25.0° west of north, and the resultant of these two pulls is 460.0 N directly northward. Use vector components to find the magnitude of each of these pulls and the direction of the smaller pull.

Solution

Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be the two forces, and let the first one be larger in magnitude: $F_1 = 2F_2$.



Draw the triangle corresponding to the vector magnitudes.



Use the law of cosines.

$$F_2^2 = F_1^2 + 460^2 - 2(F_1)(460)\cos 25°$$

Replace F_1 with $2F_2$.

$$F_2^2 = (2F_2)^2 + 460^2 - 2(2F_2)(460)\cos 25°$$

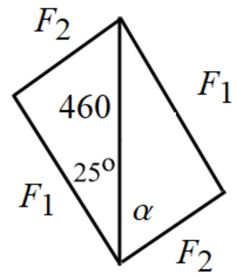
Solve for F_2.

$$0 = 3F_2^2 - 1840(\cos 25°)F_2 + 460^2$$

Use the quadratic formula.

$$F_2 = \frac{1840(\cos 25°) \pm \sqrt{[1840(\cos 25°)]^2 - 4(3)(460^2)}}{2(3)} \approx \{393, 179\}$$

Determine the direction of the weaker force by finding α in the figure below.



Use the law of cosines again for the triangle on the right.

$$F_1^2 = F_2^2 + 460^2 - 2(F_2)(460)\cos\alpha$$

Solve for α.

$$\cos\alpha = \frac{F_2^2 + 460^2 - F_1^2}{2(F_2)(460)}$$

$$\alpha = \cos^{-1}\left[\frac{F_2^2 + 460^2 - F_1^2}{2(F_2)(460)}\right]$$

Therefore, if $F_2 \approx 179$ N, then $F_1 = 2F_2 \approx 359$ N and $\alpha \approx 45.8°$; if $F_2 \approx 393$ N, then $F_1 = 2F_2 \approx 786$ N and $\alpha \approx 134°$. The figure below illustrates this second case more accurately.